RESTRICTED CIRCULATION

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Relationships between day, night and 24-hourly averages of certain Penman variables

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1. Introduction

When estimating water losses from vegetated surfaces the combination formula

$$\lambda E = \frac{\Delta' (R_N - G) + \varrho c_p \delta q / r_a}{\Delta' + \frac{c_p}{\lambda}(1 + r_s / r_a)} \qquad (1)$$

is usually used, where

r_a = aerodynamic resistance of canopy
r_s = bulk stomatal resistance of canopy
R_N = net radiation at the top of the vegetation
G = soil heat flux
$\Delta' = \partial q_w / \partial T$
($q_w(T)$= saturation specific humidity at temperature T)
δq = specific humidity deficit at height z (Temp T_z)

The specific humidity is related to the vapour pressure e and pressure p by

$$q = \frac{0.622e}{p - 0.378e}$$

R_N can be estimated in terms of n/N, T and e where

n = measured duration of bright sunshine
N = maximum possible duration of bright sun.

r_a can be expressed in terms of wind speed u and atmospheric stability. If the crop cover is dense the combination formula is used by day to calculate transpiration, and night-time applications of the formula estimate the rate of evaporation of intercepted water or the rate of condensation of water onto the crop (r_s=o). However the average wind speed is usually higher by day than at night and so for accurate applications of Eq(1) in cases when only the 24-hour run of wind is available it is necessary to estimate the ratio of the average wind by day or night to the 24-hour average. Similarly the departures of day and night time averages of T and δq from the 24-hour average values are required also.

2. Method

Differences between day and night time values of u, T and δq are expected to be related to atmospheric stability which in turn is a function of the vertical flux of buoyancy and the shear stress. However a rough measure of stability is n/N (n = hours of bright sunshine, N = maximum possible hours of sunshine) provided day-to-day variations of wind speed are ignored. This sunshine ratio was used therefore to classify the results.

Data used were 10 m winds and screen observations for Prestwick over the 10 year period 1968-1977.

A computer program extracted hourly readings of T, u and e, together with n. A value for the saturation vapour pressure

$$e_s = \exp(19.0785 - 4098.025/(T+237.3)) \qquad (2)$$

was calculated for each value of T and hence a value of $e_s - e$ was obtained for each hourly observation time. A subroutine calculated a value of N for each day. The time of sunrise and sunset taken to the nearest hourly observation time was also calculated for each day.

Averages of T, u, $e_s - e$ ($= \delta e$) were then calculated over the whole 24-hour period. (Seaton and Wales-Smith showed in Evap. Memo No 16 that the mean of daily max and min temperature and the 0900 vapour pressure are also satisfactory approximations to the 24-hour mean temperature and vapour pressure). The averages of T, u, δe for the period sunrise-sunset and the night period were then calculated. If $T_M, u_M, \delta e_M$ are the means over the 24-hour period, $T_D, u_D, \delta e_D$ over sunrise-sunset and $T_N, u_N, \delta e_N$ means over the night time period then the program calculated and listed values for n/N, u_D/u_M, u_N/u_M, $T_D - T_M$, $T_N - T_M$, $\delta e_D - \delta e_M$ and $\delta e_N - \delta e_M$. The data were then analysed as described below.

3. Temperature Averages

a) Daytime. Graphs of $T_D - T_M$ against n/N were plotted for each month of the year. Although there was a fairly wide scatter the observations could be fitted approximately by a straight line. Figure 1 shows observations for June. The best straight lines were obtained for each graph (by eye) and it was found that there was a progression of gradients and intercept points of the lines through the year. This is illustrated in Figure 2 where the gradients get progressively steeper and the intercept points higher, peaking in July/August and then falling off again in later months. $T_D - T_M$ is generally positive showing that daytime temperatures are generally higher than the 24-hour average and the greater the amount of sunshine the greater this effect. The gradients and intercepts of the lines are summarised in Table 1.

Table 1 Slopes and Intercepts of regression line for $T_D - T_M$

Month	Gradient	Intercept
Jan	0.895	0.13
Feb	1.242	0.16
Mar	1.410	0.35
April	1.411	0.38
May	1.832	0.38
June	1.842	0.42
July	1.857	0.42
Aug	1.926	0.48
Sept	1.905	0.23
Oct	1.484	0.23
Nov	1.305	0.09
Dec	0.821	0.05

Graphs of gradient against month number and intercept against month number were then drawn and 3rd order regression analysis was used to obtain a correlation between gradient of graph (or intercept) and month number. Figure 3 shows that a cubic expression provides a very good fit to the measured values. The calculated regressions are

$$\text{Gradient} = (GR)_{DT} = 0.74 + 0.19M + 0.013\,M^2 - 0.0023\,M^3 \qquad (3)$$

$$\text{Intercept} = (I)_{DT} = -0.058 + 0.17M - 0.017\,M^2 + 0.00027\,M^3 \qquad (4)$$

where M = month number.

Hence an expression for obtaining a daily average temperature from a 24-hour one is

$$T_D = T_M + (GR)_{DT} \cdot (n/N) + (I)_{DT} \qquad (5)$$

b) Night-time. Graphs of $T_N - T_M$ against n/N were drawn for the months Jan, April, July, October. Correlating $T_N - T_M$ with n/N requires the assumption that cloud cover at night is the same as that during the day which is not necessarily true. However these data showed a similar scatter to Figure 1 and could be fitted approximately by straight lines. $T_N - T_M$ was mostly negative and negatively correlated with n/N. The lines fitted to these graphs showed a progression in gradients and intercepts similar to those for $T_D - T_M$. These are summarised in table 2.

Table 2 Slopes and intercepts for $T_N - T_M$

Month	Gradient	Intercept
Jan	-0.7	-0.16
April	-1.83	-0.48
July	-2.089	-0.575
Oct	-1.684	-0.45

Since the yearly progression was similar to that found before it is reasonable to fit a cubic expression to graphs of gradient/month or intercept/month as before. The expressions thus obtained were

$$(GR)_{NT} = -0.094 - 0.67M + 0.064M^2 - 0.0013M^3 \quad (6)$$

$$(I)_{NT} = -0.0025 - 0.17M + 0.013M^2 - 0.000031M^3 \quad (7)$$

These curves were of a similar shape to the others as expected. A night average temperature can thus be calculated using

$$T_N = T_M + (GR)_{NT} \cdot (n/N) + (I)_{NT} \quad (8)$$

4. Saturation Vapour Pressure Deficit Averages

a) Daytime. The analysis of the data was carried out in the same way as before, graphs of $\delta e_D - \delta e_M$ against n/N being drawn for the months January, April, July and October. These were again fitted roughly by straight lines whose gradients and intercept points again showed a progression through the year. (Table 3)

Table 3 Slopes and intercepts for $\delta e_D - \delta e_M$ and $\delta e_N - \delta e_M$

Month	Gradient (Day)	Intercept	Gradient (Night)	Intercept
Jan	.410	0.04	-0.2947	-0.09
April	.989	0.20	-1.326	-0.34
July	1.589	0.35	-1.768	-0.52
Oct	1.347	0.28	-1.226	-0.38

Cubic expressions were fitted to the graphs of gradient/month and intercept/month, producing curves similar in shape to those found already. The equations used were

$$(GR)_{De} = 0.37 - 0.021M + 0.065M^2 - 0.0053M^3 \quad (9)$$

$$(I)_{De} = 0.021 + 0.0056M + 0.015M^2 - 0.0013M^3 \quad (10)$$

Thus

$$\delta e_D = \delta e_M + (GR)_{De} \cdot (n/N) + (I)_{De} \quad (11)$$

is a method of evaluating a daily value for δe from a 24 hour average. Again increasing n/N increases the effect, $\delta e_D - \delta e_M$ being generally positive.

b) Night-time. Graphs of $\delta e_N - \delta e_M$ against n/N were drawn for the same four months and the results of fitting straight lines to these are shown in Table 3. The expressions for gradient and intercept

$$(GR)_{Ne} = 0.11 - 0.41M + 0.0035M^2 + 0.0024M^3 \quad (12)$$

$$(I)_{Ne} = -0.034 - 0.043M - 0.015M^2 + 0.0015M^3 \quad (13)$$

were found to fit these data. $\delta e_N - \delta e_m$ was negative and again negatively correlated with n/N. An equation for calculating a night time average value for δe is therefore

$$\delta e_N = \delta e_m + (GR)_{Ne} \cdot (n/N) + (I)_{Ne} \quad (14)$$

5. Wind Averages

a) Daytime. Graphs of u_D/u_m against n/N were drawn for the months Jan, April, July and Oct. These data showed as expected a considerably smaller scatter than the earlier graphs and could again be fitted roughly by straight lines. The lines had a much shallower gradient showing a smaller dependance on n/N and there was also a much smaller progression of gradients and intercept points through the year as shown in Table 4.

Table 4 Slopes and intercepts for u_D/u_m and u_N/u_m

Month	Gradient (Day)	Intercept	Gradient (Night)	Intercept
Jan	0.0737	1.0	-0.1105	0.98
April	0.121	1.04	-0.189	0.94
July	0.179	1.10	-0.221	0.88
Oct	0.116	1.05	-0.168	0.93

On the basis of the previous results it is reasonable to expect cubic expressions to fit the graphs of gradient/month and intercept/month. The expressions used for the curves which are a similar, though shallower, shape to the others were

$$(GR)_{Du} = 0.045 + 0.026M - 0.00062M^2 - 0.00011M^3 \quad (15)$$

$$(I)_{Du} = 0.98 + 0.015M + 0.0014M^2 - 0.0002M^3 \quad (16)$$

Thus the result

$$u_D = u_m ((GR)_{Du} \cdot (n/N) + (I)_{Du}) \quad (17)$$

was obtained

b) Night-time Graphs of u_N/u_m against n/N were drawn for the same four months as before. These showed a similar scatter, shallowness of gradient, and progression of gradients and intercept points through the year as the graphs of u_D/u_m against n/N. u_N/u_m was generally less than 1 and decreased with increasing n/N. The results of fitting straight lines to these graphs are given in table 4. The cubic expressions fitted to graphs of gradient/month and intercept/month were

$$(GR)_{Nu} = -0.08 - 0.031M - 0.000099M^2 + 0.00023M^3 \quad (18)$$

$$(I)_{Nu} = 0.99 - 0.012M - 0.0022M^2 + 0.00026M^3 \quad (19)$$

The equation

$$u_N = u_m ((GR)_{Nu} \cdot (n/N) + (I)_{Nu}) \quad (20)$$

could thus be used to obtain a value for the night-time average wind speed from a 24-hour run of wind measurement.

6. Discussion and Conclusions

These results show that it is possible to calculate day and night averages of T, u and

δe when only "daily" values are available (mean of max and min temperatures, 0900 vapour pressure and 24 hour run of wind). However the regression equations conceal considerable scatter in the basic data caused in part by synoptic variability and the use of n/N as a measure of atmospheric stability. 10 m winds have been used in this study and for practical applications these may have to be converted to 2 m winds at which height the differences between day and night averages will be greater. The work has only involved data from one station and needs verifying for other stations around the country because the regression coefficients are unlikely to be universal. A computer program could be developed to fit straight lines automatically to the data and then cubic expressions to their gradients and intercepts as functions of month number without the need to print out any raw data. Conversion of hourly 10 m wind speeds to 2 m could be included at the same time.

Any comments or suggestions on this paper should be referred to the author.

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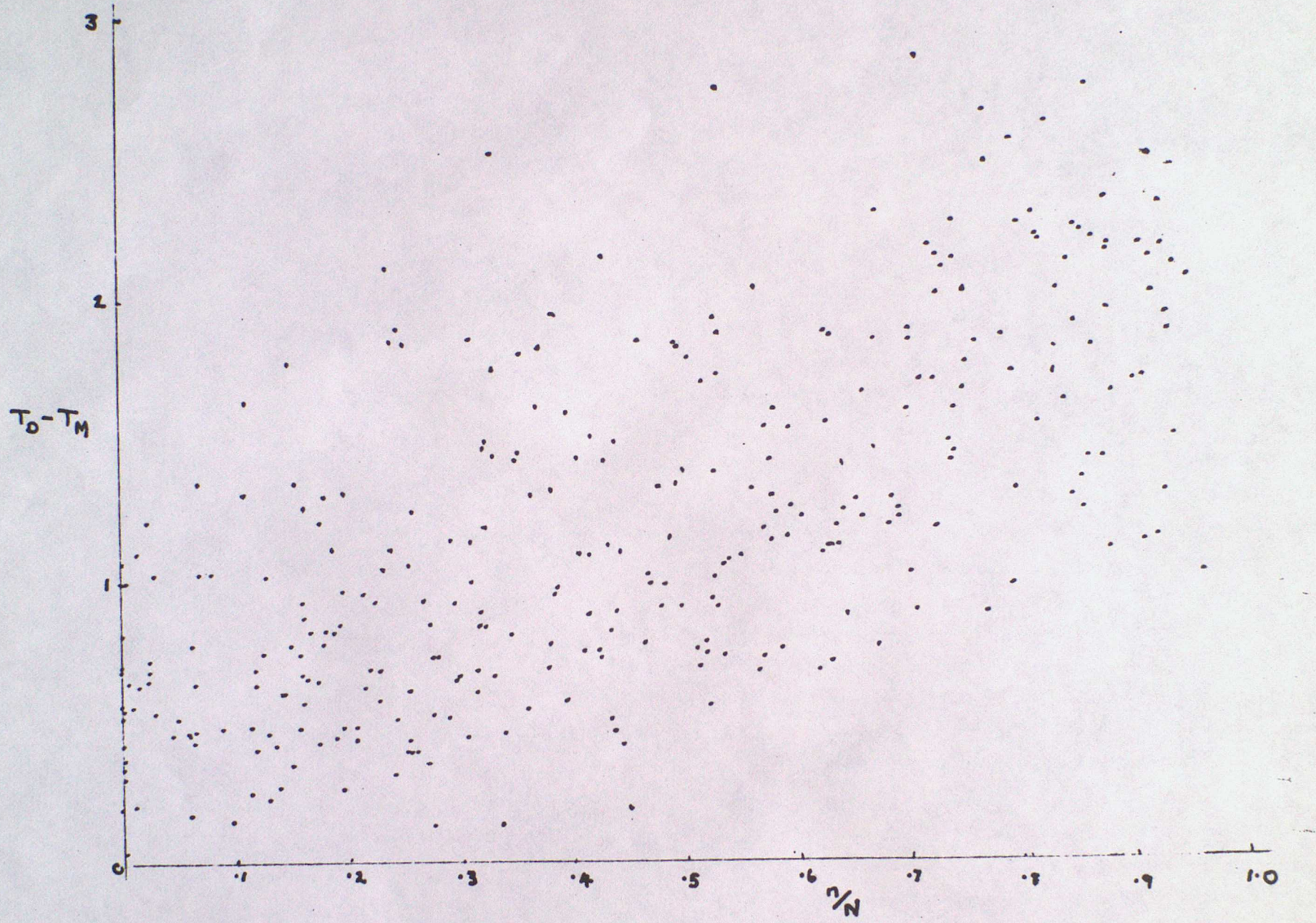

FIGURE 1 $T_D - T_M$ AS A FUNCTION OF n/N FOR JUNE

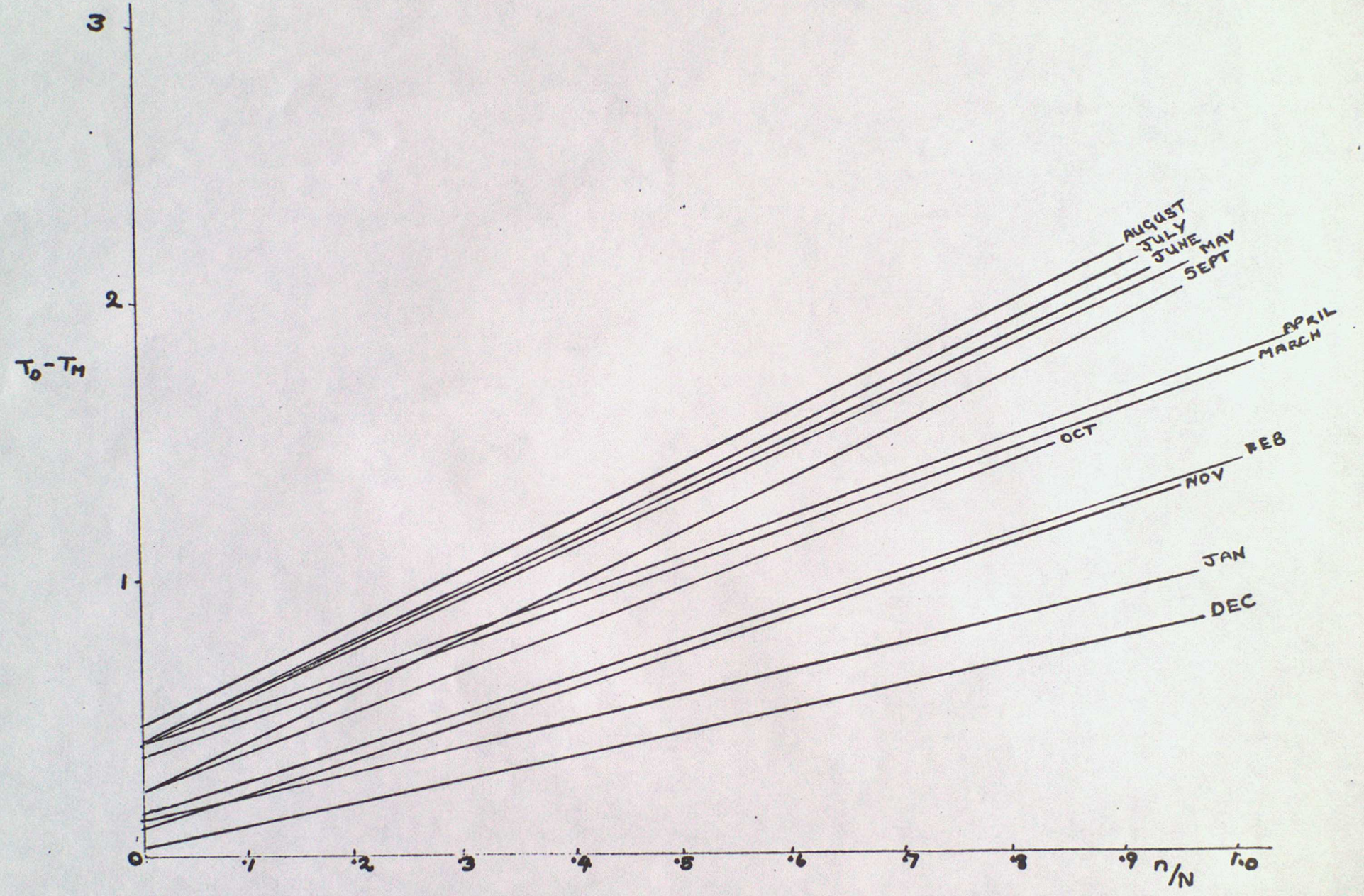

FIGURE 2 PROGRESSION OF LINES THROUGH THE YEAR FOR $T_D - T_M$

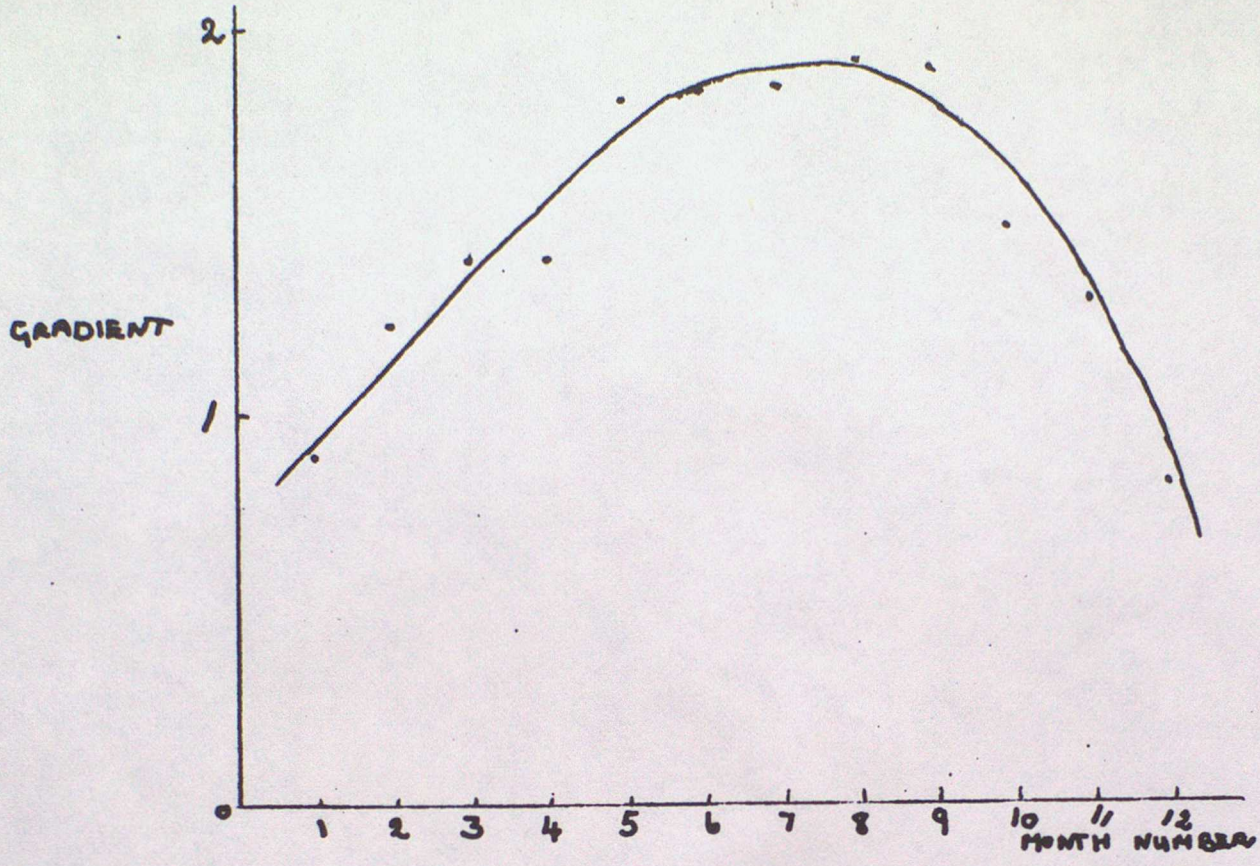

FIGURE 35) GRADIENT OF $T_D - T_M$ LINES AS A FUNCTION OF MONTH NUMBER

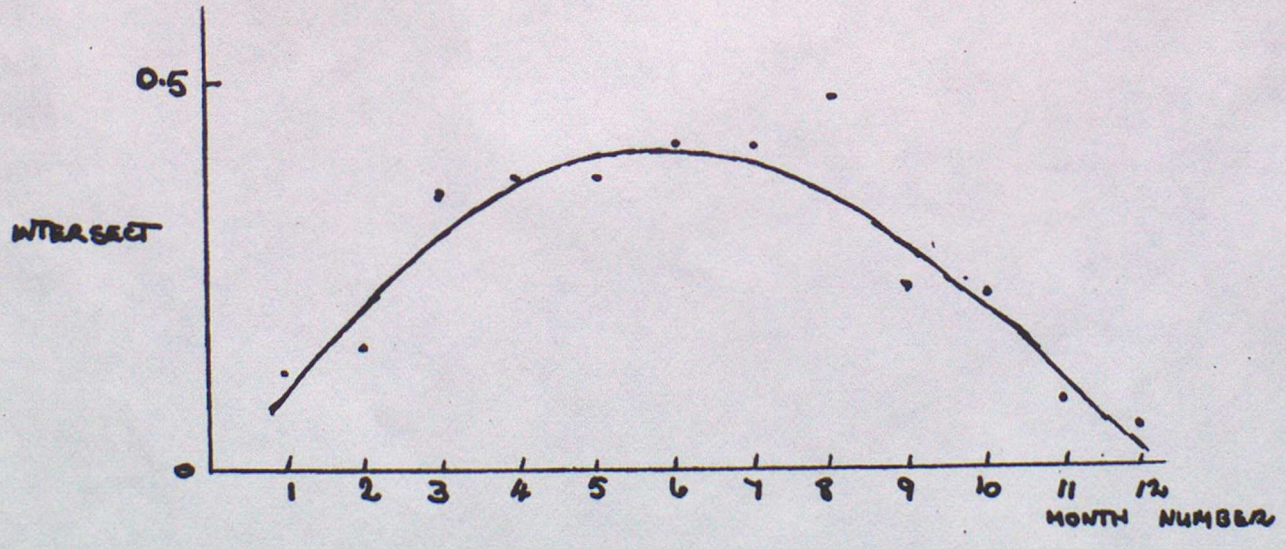

FIGURE 36) INTERCEPT POINTS OF $T_D - T_M$ LINES AS A FUNCTION OF MONTH NUMBER